“FINITE DIFFERENCE APPROACH TO WAVE GUIDE MODES COMPUTATION”

Ing.Alessandro Fanti

Electromagnetic Group
Department of Electrical and Electronic Engineering
University of Cagliari
Piazza d’Armi , 09123 Cagliari, Italy

SUMMARY

- Introduction
- Finite Difference approach to the mode evaluation for an elliptic waveguide. The use of 2D elliptical grid allows to take exactly into account the elliptical boundary. As a consequence, we get an high accuracy,with a reduced computational burden, since the resulting matrix is highly sparse;
- Standard Finite difference computation of waveguide modes requires two different grids, one for TE and another for TM modes, because the boundary conditions are different. We propose and assess here use of a single grid.

SUMMARY

- A finite-difference technique to compute Eigenvalues and mode distribution of non standard waveguide (and aperture) is presented. It is based on a mixed mesh (cartesian-polar) to avoid discretization of curved edges, and is able to give an accuracy comparable to FEM techniques with a reduced computational burden.
- A new general scheme for the FD approximation of the Laplace operator, based on a non-regular discretization, is discussed here. It allows to take into account in the FD scheme the boundary conditions, and therefore allows to use the exact shape of the boundary. As a consequence, the field distribution details can be more accurately modeled.

INTRODUCTION

An accurate knowledge of the cut-off frequency and field distribution of waveguide modes is important in many waveguide problems.
The same type of information is necessary in the analysis with the method of moments (MOM) of thick-walled apertures. Indeed, these apertures can be considered as waveguide, and the modes of these guides are the natural basis functions for the problem.
Apart from some simple geometries, mode computation cannot be done in closed forms, so that suitable numerical techniques must be used. A popular technique for cut-off frequency and field distribution evaluation is Finite Difference (FD), .i.e, direct discretization of the eigenvalue problem. This allows a simple and very affective evaluation, also because the problem is reduced to the computation of the eigenvalues and eigenvectors of an highly sparse matrix

INTRODUCTION

The standard four-point FD approximation of the Laplace operator, however, cannot be used for more complex geometry since it require a regular (rectangular) discretization grid, and therefore a boundary with all sides parallel to the rectangular axes. Therefore circular and elliptic boundaries are typically replaced by stair case approximation.

Aim of this presentation is to develop, and assess, a general scheme for the FD approximation of the Laplace operator, based on a regular polar and elliptic grid.

DESCRIPTION OF THE TECNIQUE

Standard FD discretization in Cartesian coordinates for a rectangular cell :

leads to the approximation of the Laplace oparator

$$\nabla_t^2 \varphi\big|_0 = \frac{1}{\Delta x^2 \cdot \Delta y^2} \cdot \left[\Delta y^2 \cdot \varphi_1 + \Delta x^2 \cdot \varphi_4 + \Delta y^2 \cdot \varphi_3 + \Delta x^2 \cdot \varphi_2 - 2 \cdot \left(\Delta x^2 + \Delta y^2\right) \cdot \varphi_0\right]$$

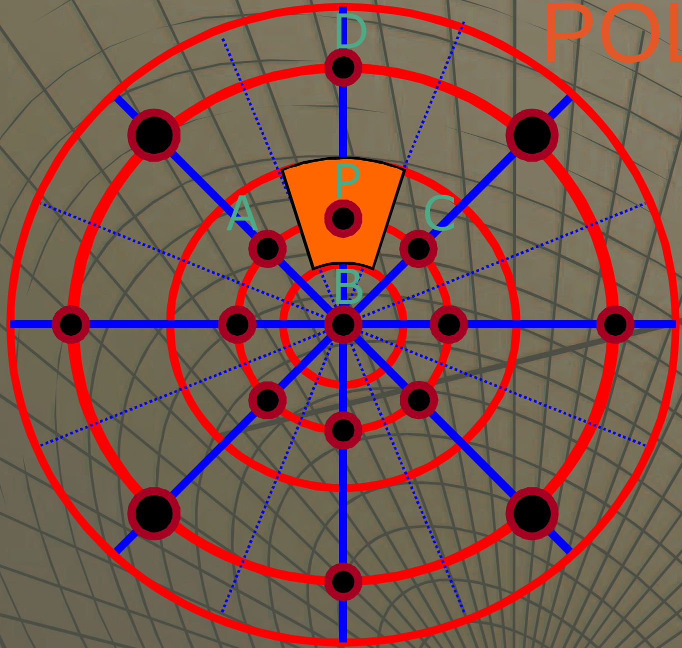
DESCRIPTION OF THE TECNIQUE POLAR FRAMEWORK

- Let use consider a circular waveguide. Both TE and TM modes can be found from a suitable scalar eigenfunction ϕ , solution of the Helmothz equation:

$$\nabla_t^2 \varphi = -k_t^2 \varphi \qquad (1)$$

with the boundary condition $\frac{\partial \varphi}{\partial n} = 0$ for TE mode, $\varphi = 0$ for TM mode

DESCRIPTION OF THE TECNIQUE
POLAR FRAMEWORK



- Consider the cell around point P enclosed points ACBD. Remember the form of the Laplacian in polar coordinates:

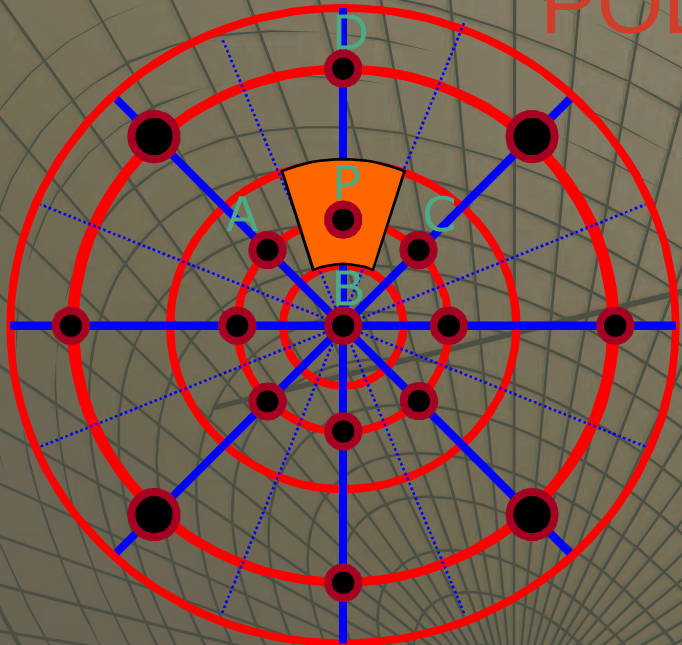
$$\nabla^2\varphi = \frac{1}{r_p^2}\cdot\left(\frac{\partial^2\varphi}{\partial\alpha^2} + r_P\cdot\frac{\partial}{\partial r}\left(r_P\cdot\frac{\partial\varphi}{\partial r}\right)\right) = \frac{1}{r_p^2}\cdot\frac{\partial^2\varphi}{\partial\alpha^2} + \frac{1}{r_p}\cdot\frac{\partial\varphi}{\partial r}|_P + \frac{\partial^2\varphi}{\partial r^2}|_P \qquad (2)$$

Using a second order Taylor approximation for points A and C, and summing:

$$\varphi_A = \varphi_P + \frac{\partial\varphi}{\partial\alpha}|_P\cdot(-\Delta\alpha) + \frac{1}{2}\frac{\partial^2\varphi}{\partial\alpha^2}|_P\cdot(-\Delta\alpha)^2 \qquad \varphi_C = \varphi_P + \frac{\partial\varphi}{\partial\alpha}|_P\cdot(\Delta\alpha) + \frac{1}{2}\frac{\partial^2\varphi}{\partial\alpha^2}|_P\cdot(\Delta\alpha)^2$$

$$\frac{\partial^2\varphi}{\partial\alpha^2}|_P = \frac{1}{(\Delta\alpha)^2}\cdot(\varphi_A + \varphi_C - 2\varphi_P)$$

DESCRIPTION OF THE TECNIQUE POLAR FRAMEWORK



- using the same procedure for points B and D we get:

$$\frac{\partial^2 \varphi}{\partial r^2}\Big|_P = \frac{1}{(\Delta r)^2} \cdot (\varphi_B + \varphi_D - 2\varphi_P)$$

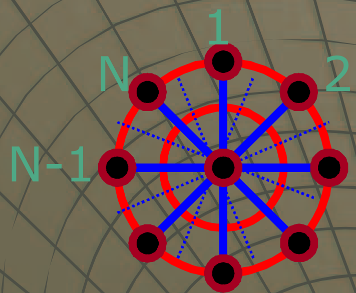
- and

$$\frac{\partial \varphi}{\partial r}\Big|_P = \frac{\varphi_B - \varphi_D}{2 \cdot \Delta r}$$

The approximation of the laplacian becomes:

$$\nabla \varphi_t^2\Big|_P = \frac{1}{r_p^2(\Delta\alpha)^2}\cdot\varphi_A + \left(\frac{1}{2r_p\Delta r} + \frac{1}{(\Delta r)^2}\right)\cdot\varphi_B + \frac{1}{r_p^2(\Delta\alpha)^2}\cdot\varphi_C + \left(\frac{1}{(\Delta r)^2} - \frac{1}{2r_p\Delta r}\right)\cdot\varphi_D - \left(\frac{2}{(\Delta r)^2} + \frac{2}{r_p^2(\Delta\alpha)^2}\right)\cdot\varphi_P$$

CENTER POINT



For the center point we integrate (1) over a discretization cell

$$\int \nabla_t^2 \varphi dS = -k_t^2 \int \varphi dS$$

Use of Gauss Theorem gives: $\int_{\Gamma_F} \nabla_t^2 \varphi \cdot i_n dl = -k_t^2 \int_{S_F} \varphi dS$

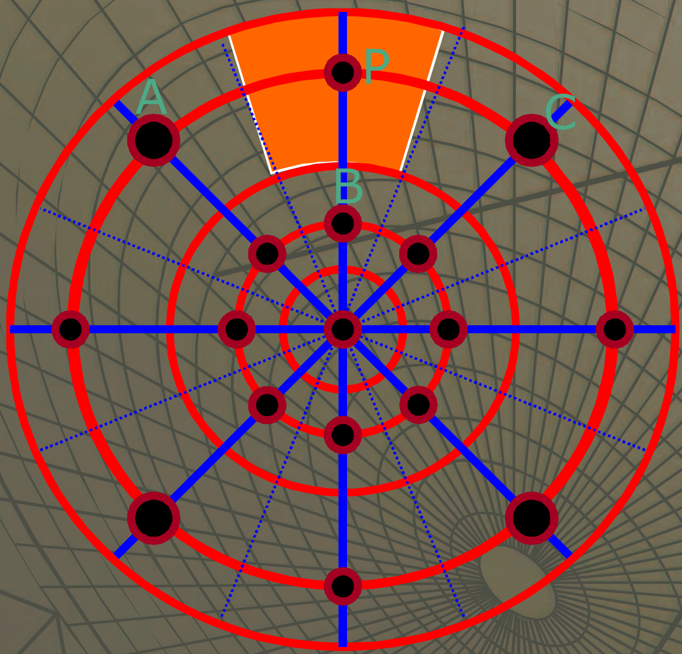
i.e $\int_{\Gamma_F} \frac{\partial \varphi}{\partial n} \cdot dl = -k_t^2 \varphi$ (3) where Γ_F is the cell boundary,

S_F is the cell surface and φ is the evaluated at the discretization node.

$$\nabla \varphi_t^2 \Big|_P = \left(\frac{1}{\pi \cdot \left(\frac{\Delta r}{2} \right)^2} \right) \cdot \sum_{q=1}^{N} \frac{(\varphi_q - \varphi_P)}{\Delta r} \cdot \frac{\Delta r}{2} \cdot \Delta \alpha \qquad (4)$$

BOUNDARY POINT

- For TE mode $\dfrac{\partial \varphi}{\partial r} = 0$



Then:

$$\nabla \varphi_t^2\Big|_P = \frac{1}{r_p^2 (\Delta \alpha)^2} \cdot \varphi_A + \left(\frac{1}{2 r_p \Delta r} + \frac{1}{(\Delta r)^2} \right) \cdot \varphi_B + \frac{1}{r_p^2 (\Delta \alpha)^2} \cdot \varphi_C - \left(\frac{1}{(\Delta r)^2} + \frac{1}{2 r_p \Delta r} + \frac{2}{r_p^2 (\Delta \alpha)^2} \right) \cdot \varphi_P$$

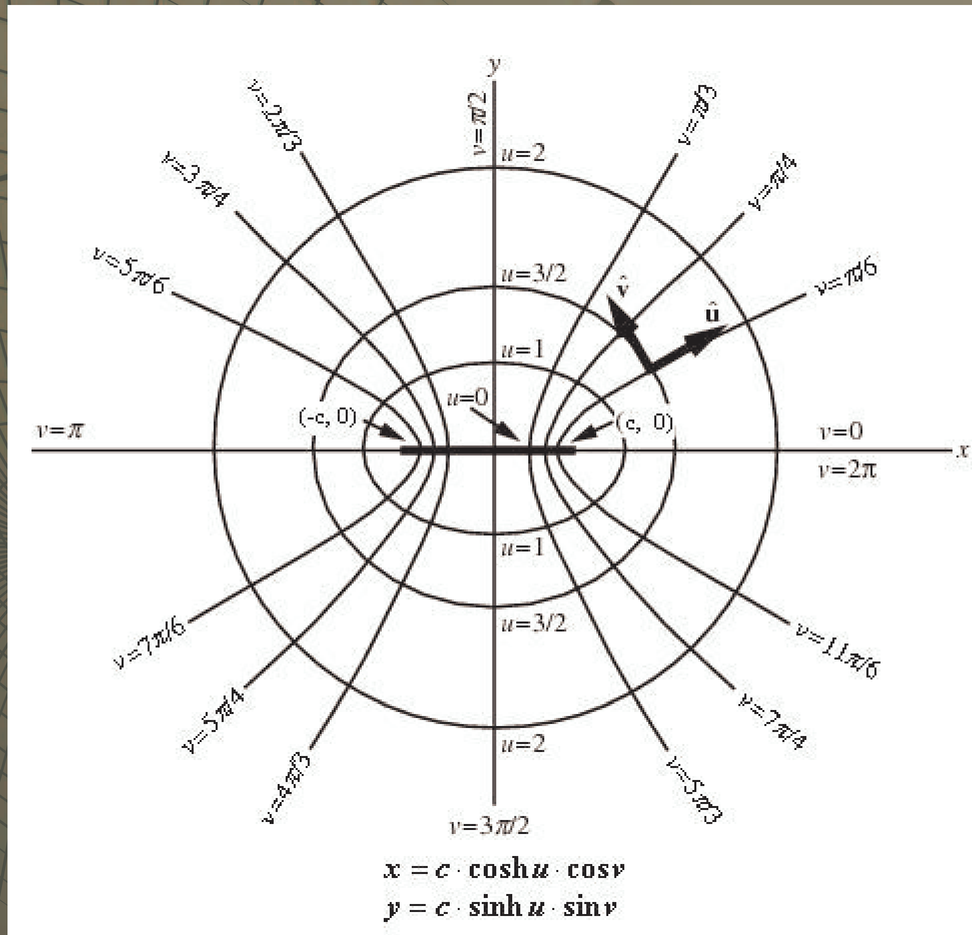
NUMERICAL RESULT

COMPARISON BETWEEN OUR FD CODE AND ANALITIC RESULTS FOR TE MODES IN CIRCULAR GUIDE WAVE

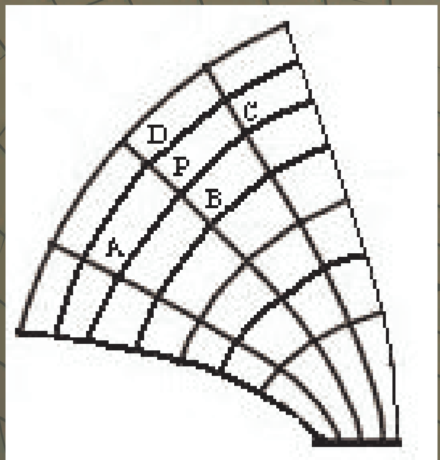
k_t (Analitic)	k_t (FIT)	k_t (Our FD code)	Relative error
0.4602	0.4604	0.4604	0.034%
0.7635	0.7633	0.7634	0.012%
0.9580	0.9572	0.9578	0.014%
1.0502	1.0493	1.0500	0.016%
1.3292	1.3284	1.3292	0.000%
1.3327	1.3313	1.3313	0.108%

DESCRIPTION OF THE TECNIQUE ELLIPTIC FRAMEWORK

Let use consider a elliptic waveguide. Both TE and TM modes can be found from a suitable scalar eigenfunction φ, solution of (1)



DESCRIPTION OF THE TECNIQUE
ELLIPTIC FRAMEWORK



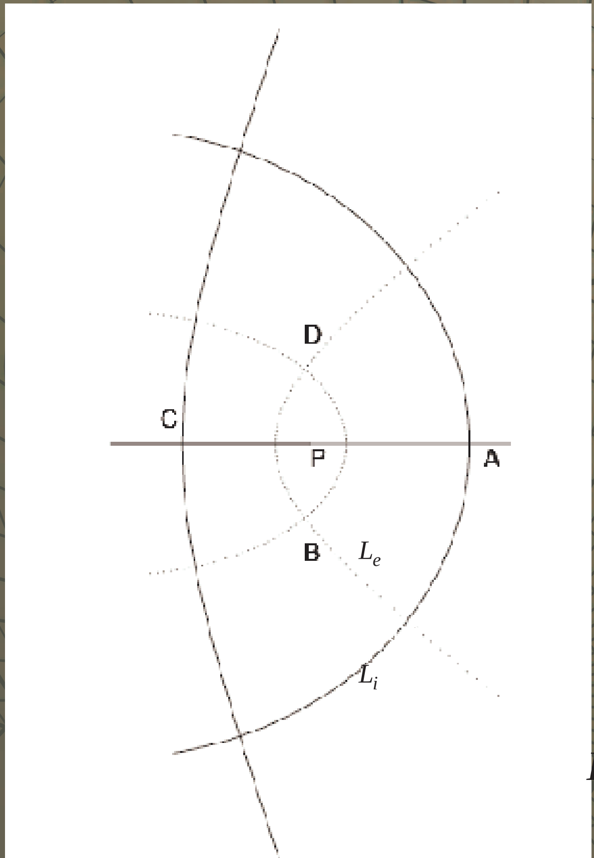
Assuming a regular spacing on the coordinate lines, with step $\Delta u, \Delta v,$ and letting $\varphi_{pq} = \varphi(p\Delta u, q\Delta v)$ the eigenvalues equation (1) can be expressed us:

$$\frac{1}{c^2 \cdot \left(\sinh^2 u + \sin^2 v\right)} \cdot \left(\frac{\partial^2 \varphi}{\partial u^2} + \frac{\partial^2 \varphi}{\partial v^2} \right) = -k_t^2 \varphi_{pq} \qquad (6)$$

the term in brackets expanded exactly as in a rectangular grid:

$$\left(\frac{\partial^2 \varphi}{\partial u^2} + \frac{\partial^2 \varphi}{\partial v^2} \right) = \left(\frac{\varphi_A}{(\Delta v)^2} + \frac{\varphi_B}{(\Delta u)^2} + \frac{\varphi_C}{(\Delta v)^2} + \frac{\varphi_D}{(\Delta u)^2} - 2\varphi_P \cdot \left(\frac{1}{(\Delta u)^2} + \frac{1}{(\Delta v)^2} \right) \right)$$

FOCUS



Finally, consider the foci of the elliptical shape grid

$$\frac{1}{S_A}\cdot\int\nabla_t\varphi\cdot i_n dl = -k_t^2\cdot\frac{1}{S_A}\cdot\int\varphi\cdot ds \cong -k_t^2\cdot\varphi_A$$

$$\frac{1}{S_A}\cdot[(\varphi_C-\varphi_P)\cdot L_E+(\varphi_A-\varphi_P)\cdot L_I]$$

S_A is the area of the cell, and L_E, L_I are half the length of the arc of the ellipse and of the arc of the hyperbola respectively.

$$h_u(u,v)=h_v(u,v)=\frac{1}{a\sqrt{\sinh^2 u+\sin^2 v}}$$

$$L_e=\int_0^{\frac{\Delta v}{2}} h_u\left(\frac{\Delta u}{2},v\right)dv \cong \frac{\Delta v}{4}\left(h_u\left(\frac{\Delta u}{2},0\right)+h_u\left(\frac{\Delta u}{2},\frac{\Delta v}{2}\right)\right)$$

$$L_i=\int_0^{\frac{\Delta u}{2}} h_v\left(u,\frac{\Delta v}{2}\right)du \cong \frac{\Delta u}{4}\left(h_v\left(\frac{\Delta v}{2},0\right)+h_v\left(\frac{\Delta u}{2},\frac{\Delta v}{2}\right)\right)$$

NUMERICAL RESULT

COMPARISON BETWEEN OUR FD CODE AND A COMMERCIAL FIT CODE FOR TE MODE IN ELLIPTIC WAVEGUIDE.

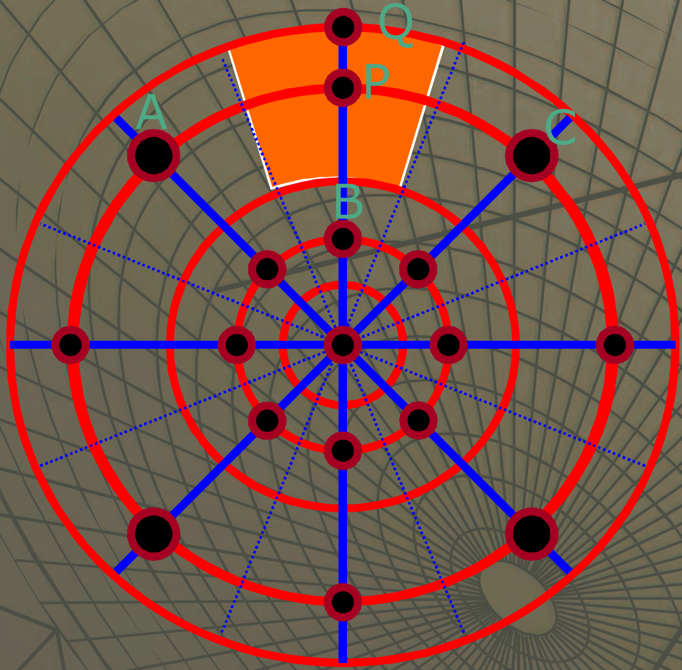
k_t (FIT)	k_t (Our FD code)	Relative error
0.2168	0.2166	0.092 %
0.3963	0.3960	0.075 %
0.4395	0.4389	0.136 %
0.5666	0.5662	0.070 %
0.5720	0.5716	0.069 %
0.7036	0.7033	0.042 %
0.7454	0.7451	0.040 %

TM MODES

Since the fundamental mode is a TE, these modes are the most interesting. TM modes can, however, be computed in a likely way, taking into account the different boundary conditions.

This was done using a grid different from TE one. This might be fine for the calculation of modes of microwave guiding structures, but for some applications (analysis by the method of moments of aperture, Mode matching) would be much more useful the TE grid. Then we explored the possibility of using a single grid for both TE and TM modes.

BOUNDARY POINT



Boundary condition: $\varphi_Q = 0$

Using a second order Taylor approximation for Q:

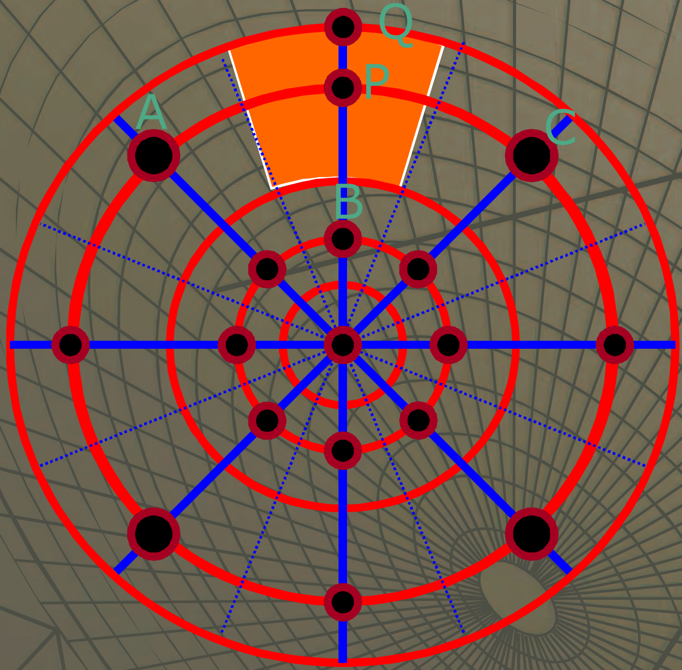
$$0 = \varphi_Q = \varphi_P + \frac{\partial \varphi}{\partial r}|_P \cdot \left(\frac{\Delta r}{2}\right) + \frac{1}{2}\frac{\partial^2 \varphi}{\partial r^2}|_P \cdot \left(\frac{\Delta r}{2}\right)^2$$

Recalling that:

$$\varphi_B = \varphi_P + \frac{\partial \varphi}{\partial r}|_P \cdot (-\Delta r) + \frac{1}{2}\frac{\partial^2 \varphi}{\partial r^2}|_P \cdot (-\Delta r)^2$$

We can compute: $\frac{\partial \varphi}{\partial r}|_P$ $\frac{\partial^2 \varphi}{\partial r^2}|_P$

BOUNDARY POINT

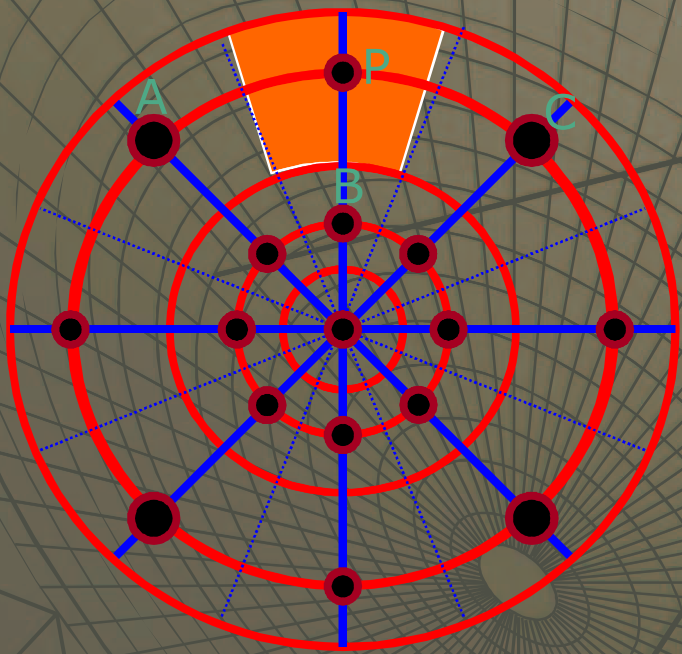


The result is:

$$\frac{\partial^2 \varphi}{\partial r^2}\Big|_P = \frac{4}{3\cdot(\Delta r)^2}\cdot(\varphi_B - 3\varphi_P)$$

$$\frac{\partial \varphi}{\partial r}\Big|_P = -\frac{(\varphi_B + 3\varphi_P)}{3\Delta r}$$

BOUNDARY POINT



- For TM modes:

$$\nabla \varphi_t^2 \Big|_P = \frac{1}{r_p^2 (\Delta \alpha)^2} \cdot \varphi_A + \left(\frac{1}{3 r_p \Delta r} + \frac{4}{3(\Delta r)^2} \right) \cdot \varphi_B + $$

$$+ \frac{1}{r_p^2 (\Delta \alpha)^2} \cdot \varphi_C - \left(\frac{4}{(\Delta r)^2} - \frac{1}{r_p \Delta r} + \frac{2}{r_p^2 (\Delta \alpha)^2} \right) \cdot \varphi_P$$

NUMERICAL RESULT

COMPARISON BETWEEN OUR FD CODE AND AND ANALITIC RESULTS FOR TM MODES IN CIRCULAR WAVE GUIDE

k_t (Analitic)	k_t (Our FD code)	Relative error
0.6013	0.6012	0.003%
0.9580	0.9579	0.005%
1.2840	1.2839	0.008%
1.3800	1.3798	0.018%
1.5950	1.5949	0.003%
1.7540	1.7535	0.029%

DESCRIPTION OF THE TECNIQUE

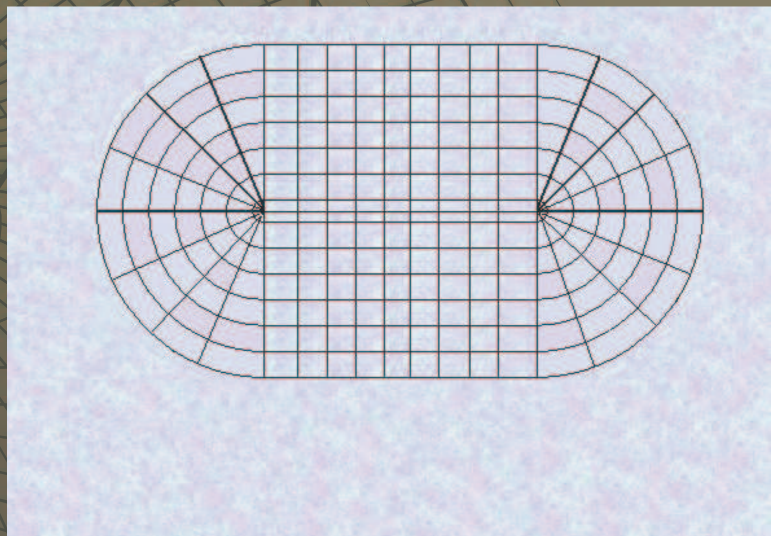
- For the all point we integrate (1) over a discretization cell

$$\int \nabla_t^2 \varphi \cdot ds = -k_t^2 \int \varphi dS$$

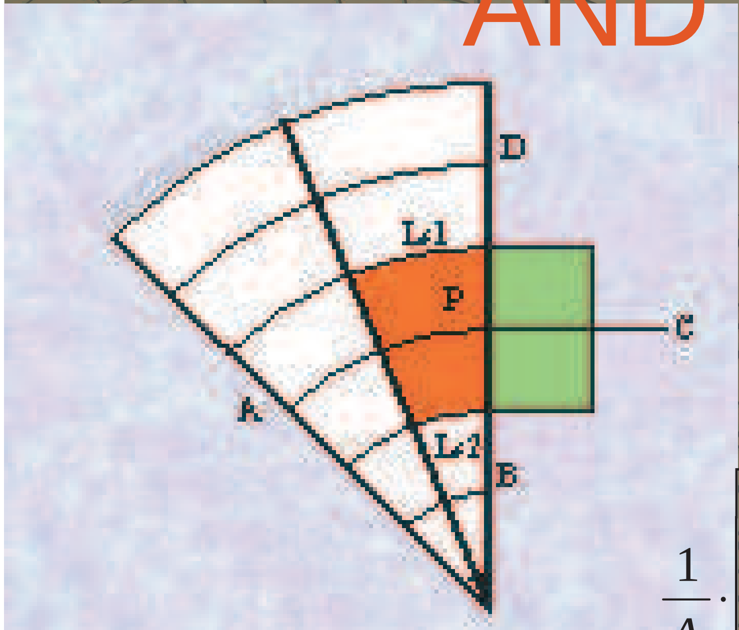
Use of Gauss Theorem gives: $\int_{\Gamma_F} \nabla_t^2 \varphi \cdot i_n dl = -k_t^2 \int_{S_F} \varphi dS$

i.e $\int_{\Gamma_F} \frac{\partial \varphi}{\partial n} \cdot dl = -k_t^2 \varphi$ (3) where φ is evaluated at the discretization node.

S_F is the cell surface and



POINTS BETWEEN CARTESIAN AND POLAR GRID

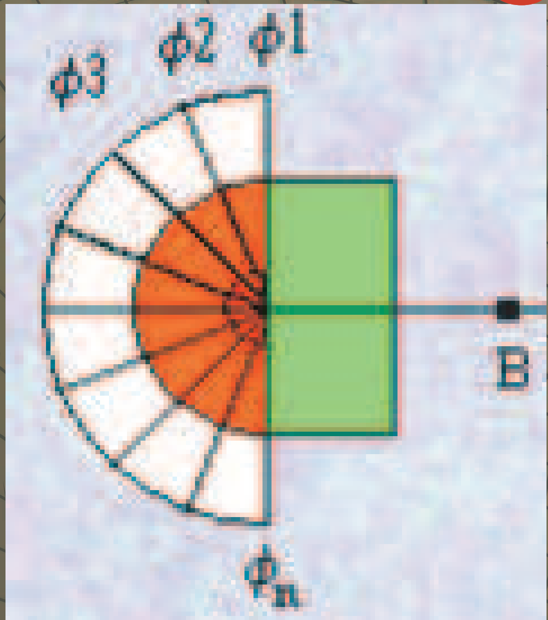


The approximation of the laplacian becomes:

$$\frac{1}{A_t}\cdot\left[\varphi_A\cdot\frac{\Delta r}{\Delta\alpha\cdot r_p}+\varphi_B\cdot\frac{\left(L_{S2}+\frac{\Delta x}{2}\right)}{\Delta r}+\varphi_C\cdot\frac{\Delta r}{\Delta x}+\varphi_D\cdot\frac{\left(L_{S1}+\frac{\Delta x}{2}\right)}{\Delta r}+\right.$$

$$\left.-\varphi_P\left(\frac{\Delta r}{\Delta\alpha\cdot r_p}+\frac{\left(L_{S2}+\frac{\Delta x}{2}\right)}{\Delta r}+\frac{\Delta r}{\Delta x}+\frac{\left(L_{S1}+\frac{\Delta x}{2}\right)}{\Delta r}\right)\right]$$

CENTER POINT



The approximation of the laplacian becomes:

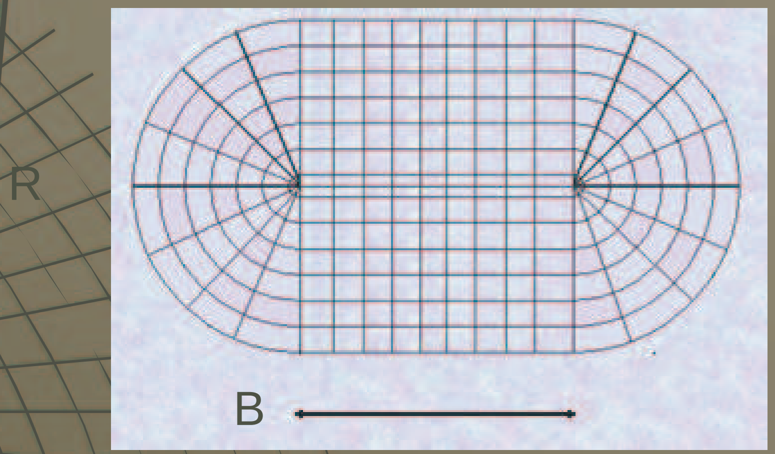
$$A_t = \pi \cdot \frac{1}{2} \cdot \left(\frac{\Delta r}{2}\right)^2 + \Delta r \cdot \frac{\Delta x}{2}$$

$$\frac{1}{A_t} \cdot \left[\left(\frac{\Delta x}{2 \cdot \Delta r} + \frac{\Delta \alpha}{2}\right) \cdot (\varphi_1 + \varphi_n) + \frac{\Delta \alpha}{2} \cdot (\varphi_2 + \varphi_3 + \ldots\ldots + \varphi_{n-2}) + \frac{\Delta r}{\Delta x} \cdot \varphi_B - \left((n-2) \cdot \frac{\Delta \alpha}{2} + 2 \cdot \left(\frac{\Delta x}{2 \cdot \Delta r} + \frac{\Delta r}{\Delta x} \right) \right) \cdot \varphi_P \right]$$

NUMERICAL RESULT

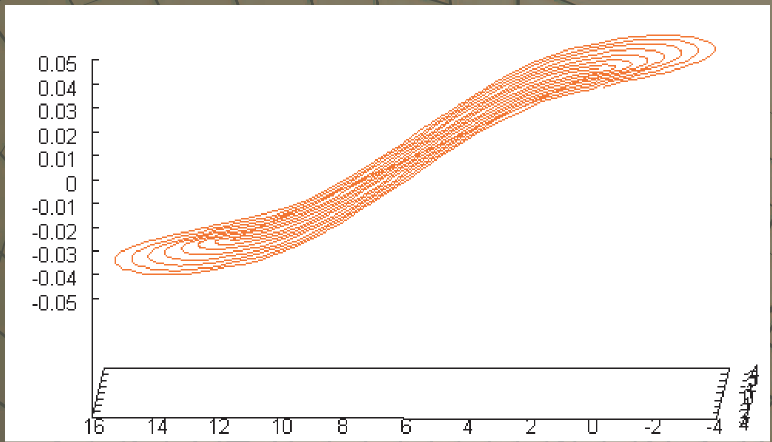
D=51p,X=51p,a=1°,r=2

ktp	kth	ktcst	ephfs%	epcst%
0.2178	0.2182	0.2182	0.1515	0.1500
0.4155	0.4154	0.4152	0.0268	0.0618
0.4289	0.4255	0.4253	0.7959	0.8438
0.5056	0.5038	0.5037	0.3507	0.3728
0.6124	0.6153	0.6143	0.4574	0.2981
0.6561	0.6521	0.6516	0.6093	0.6805
0.7785	0.7748	0.7738	0.4825	0.6084
0.8244	0.8189	0.8171	0.6695	0.8892
0.8318	0.8265	0.8255	0.6407	0.7622
0.8771	0.8722	0.8711	0.5541	0.6827

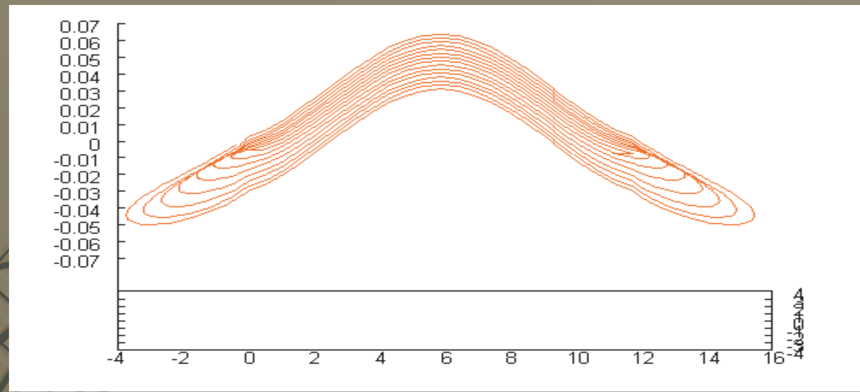


R = 2mm,B = 6.137mm

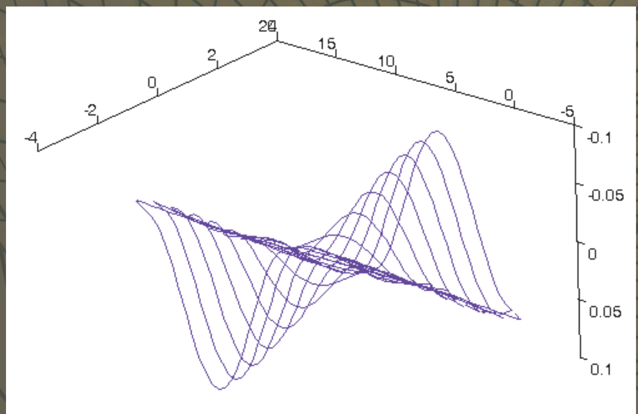
GRAPHICS



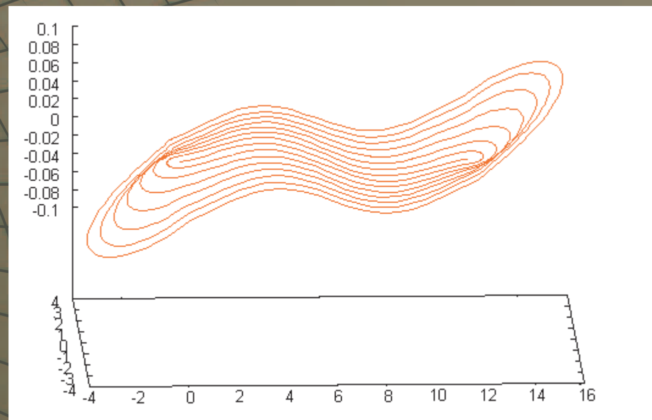
MODO FONDAMENTALE TE



I MODO SUPERIORE TE



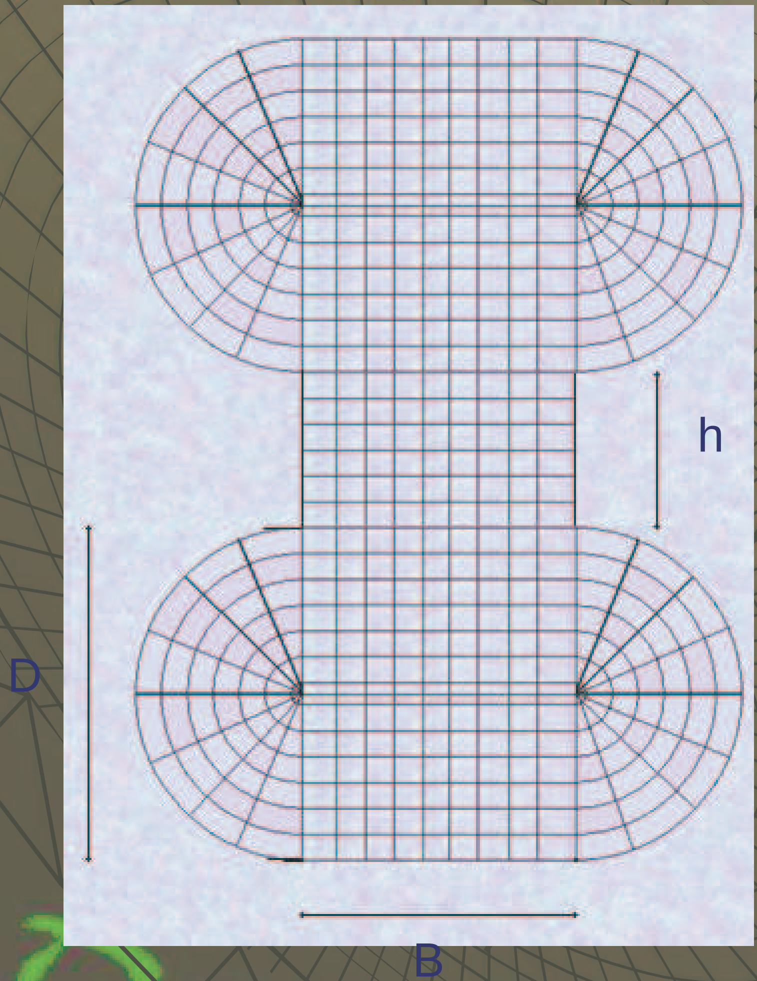
II MODO SUPERIORE TE



III MODO SUPERIORE TE

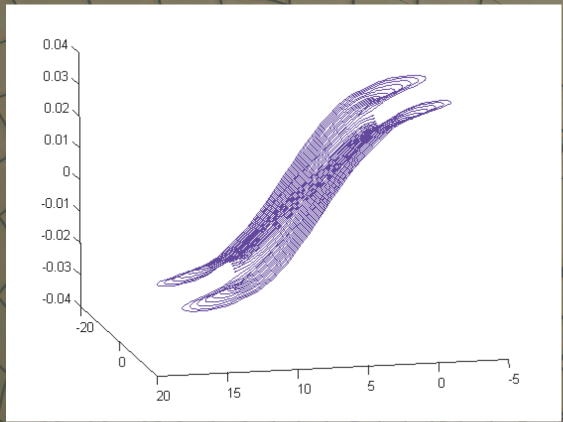
NUMERICAL RESULT

D = 4 mm,B = 6.137mm, h=2.6

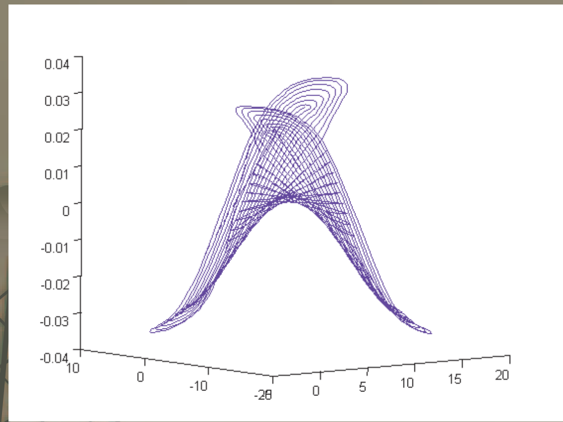


MODI	Kt (HFSS)	Kt (num)
Modo fond TE	0.2448815	0.2432115
1 modo sup TE	0.3577106	0.3560906
2 modo sup TE	0.3949076	0.3933276
3 modo sup TE	0.5460041	0.5443841
4 modo sup TE	0.585.2994	0.5837194

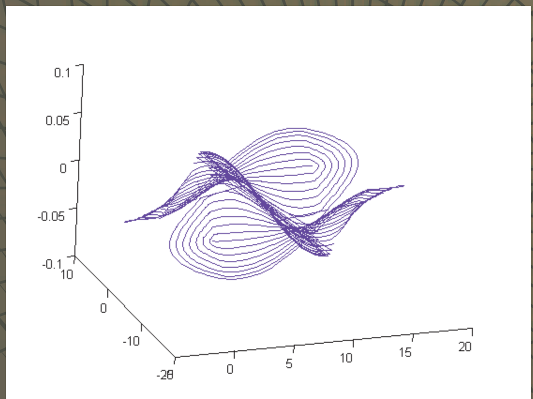
GRAPHICS



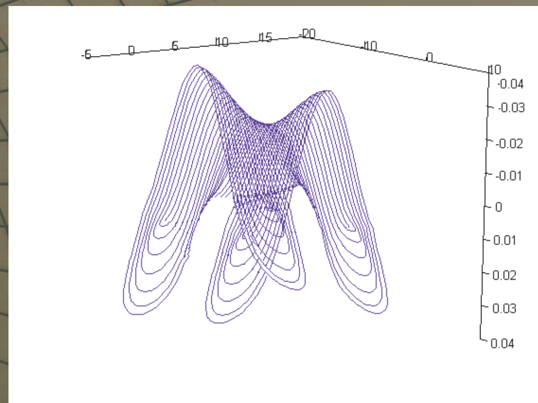
MODO FONDAMENTALE TE



I MODO SUPERIORE TE



II MODO SUPERIORE TE



III MODO SUPERIORE TE

DESCRIPTION OF THE TECNIQUE

Standard FD discretization in Cartesian coordinates for a rectangular cell :

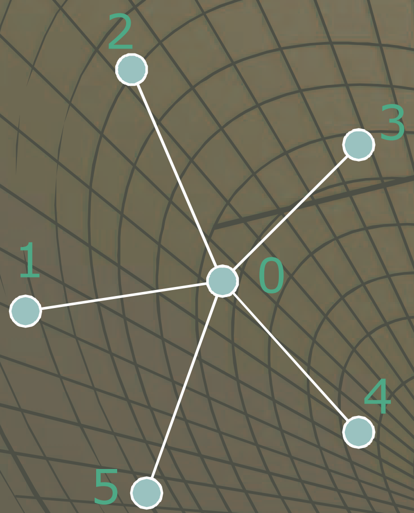
leads to the approximation of the Laplace oparator

$$\nabla_t^2 \varphi\big|_0 = \frac{1}{\Delta x^2 \cdot \Delta y^2}\cdot\left[\Delta y^2\cdot\varphi_1 + \Delta x^2\cdot\varphi_4 + \Delta y^2\cdot\varphi_3 + \Delta x^2\cdot\varphi_2 - 2\cdot\left(\Delta x^2+\Delta y^2\right)\cdot\varphi_0\right]$$

Our interest is to use irregular grids

DESCRIPTION OF THE TECNIQUE

A consider a non standard discretization



We are an looking for: $\nabla_t^2 \varphi \big|_0 = \sum_i A_i (\varphi_i - \varphi_0)$

Using a second order Taylor approximation we get:

$$\varphi_i - \varphi_0 = \frac{\partial \varphi}{\partial x}\Big|_0 \cdot \Delta x_i + \frac{\partial \varphi}{\partial y}\Big|_0 \cdot \Delta y_i + \frac{1}{2}\frac{\partial^2 \varphi}{\partial x^2}\Big|_0 \cdot \Delta x_i^{\,2} + \frac{1}{2}\frac{\partial^2 \varphi}{\partial y^2}\Big|_0 \cdot \Delta y_i^{\,2} + \frac{\partial^2 \varphi}{\partial x \partial y}\Big|_0 \cdot \Delta x_i \cdot \Delta y_i$$

where all derivatives of ϕ are computed at the sampling point, and $(\Delta x_i, \Delta y_i)$ the position of the i–th point w.r.t point 0.

DESCRIPTION OF THE TECNIQUE

Therefore $$\sum_i A_i(\varphi_i - \varphi_0) = B_1 \frac{\partial\varphi}{\partial x} + B_2 \frac{\partial\varphi}{\partial y} + B_3 \frac{\partial^2\varphi}{\partial x^2} + B_4 \frac{\partial^2\varphi}{\partial y^2} + B_5 \frac{\partial^2\varphi}{\partial x \partial y} \qquad (1)$$

The B_i are linear combination of the unknown coefficients A_i.

For example B_1 is equal to:

$$B_1 = A_1 \Delta x_1 + A_2 \Delta x_2 + A_3 \Delta x_3 + A_4 \Delta x_4 + A_5 \Delta x_5$$

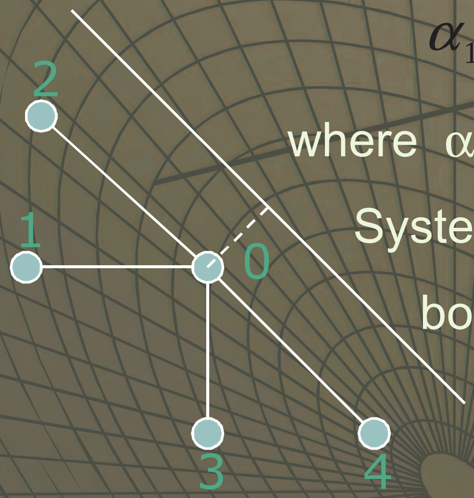
To get the Laplace operator we required

$B_1=B_2=B_5=0$ $B_3=B_4=1$ (2) which is a linear system in the A_i.

BOUNDARY POINT

For a boundary point, boundary condition δφ/δn=0 can be expressed as:

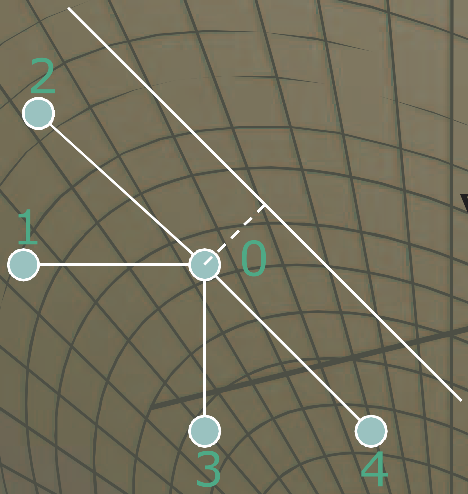
$$\alpha_1 \frac{\partial \varphi}{\partial x}\Big|_5 + \alpha_2 \frac{\partial \varphi}{\partial y}\Big|_5 = 0 \qquad (3)$$



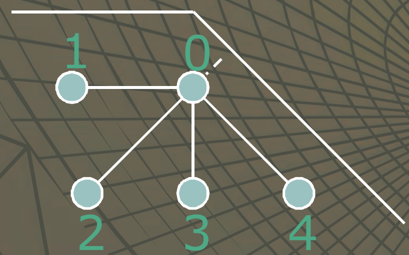
where α_1, α_2 are the component of a vector normal to the boundary.

System (2) for a boundary point is modified tuning in to account boundary condition (3).

EXAMPLE OF BOUNDARY CONDITION



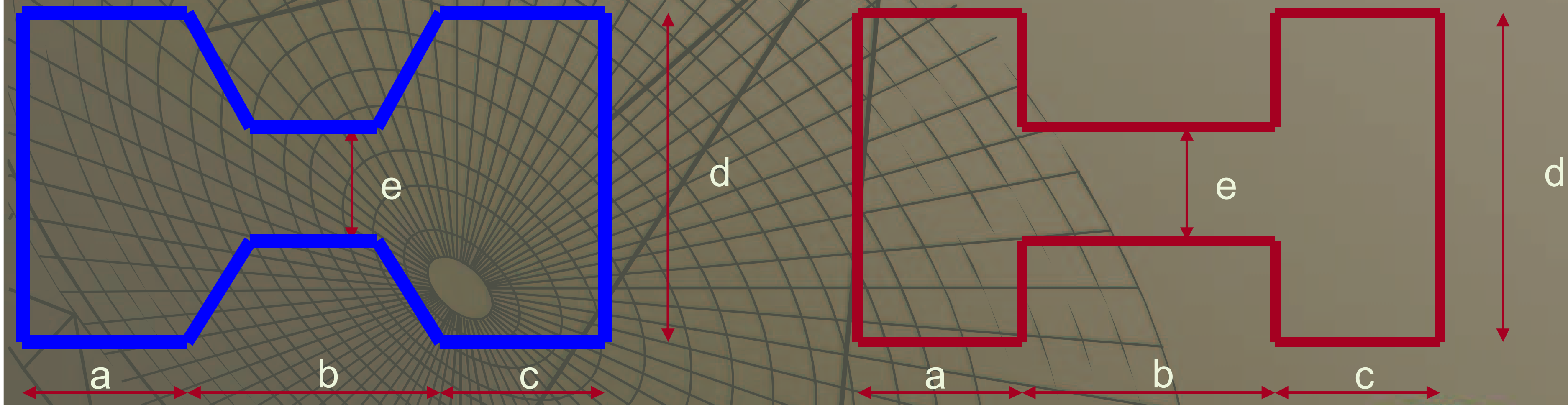
$$\nabla_t^2 \varphi\Big|_0 = (\varphi_1+\varphi_2)\frac{2}{\Delta x^2} - \varphi_0 \frac{4}{\Delta x^2}$$



$$\nabla_t^2 \varphi\Big|_0 = (\varphi_1+\varphi_4)\frac{1}{\Delta x^2} + (\varphi_2+\varphi_3)\frac{1}{2\Delta x^2} - \varphi_0 \frac{3}{\Delta x^2}$$

NUMERICAL RESULT

To assess our FD technique with variable grid, we have analyzed a ridged waveguide with trapezoidal ridges and rectangular aperture.



NUMERICAL RESULT

TE Mode of ridge waveguide trapezoidal
aperture with e=1.55 mm

TE mode	Kt (FD)	Kt (CST)
I	0.1774	0.1776
II	0.6220	0.6222
III	0.6324	0.6325
IV	0.6325	0.6326

NUMERICAL RESULT

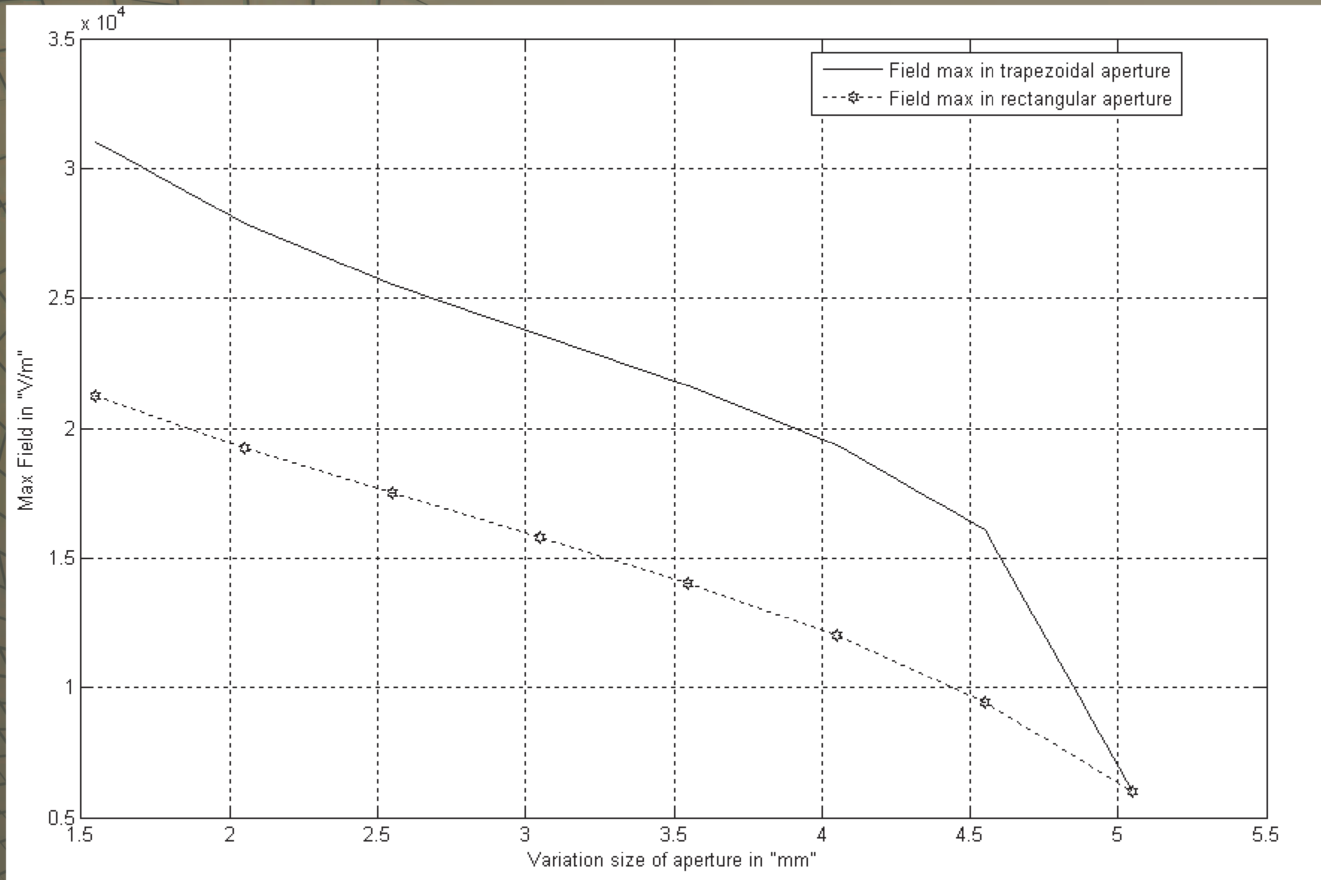
TE Mode of ridge waveguide rectangular aperture with e=2.55 mm

TE mode	Kt (FD)	Kt (CST)
I	0.2214	0.2216
II	0.5850	0.5854
III	0.6697	0.6699
IV	0.6711	0.6714

NUMERICAL RESULT

TE mode	1.55 mm	2.05 mm	2.55 mm	3.05 mm	3.55 mm	4.05 mm	4.55 mm
I	0.1774	0.1991	0.2180	0.2347	0.2496	0.2625	0.2733
II	0.6220	0.6065	0.5963	0.5889	0.5829	0.5772	0.5710
III	0.6324	0.6388	0.6450	0.6493	0.6503	0.6463	0.6367
IV	0.6325	0.6393	0.6472	0.6555	0.6638	0.6715	0.6780
TE Mode of ridge waveguide trapezoidal aperture and rectangular aperture, increases “e”							
I	0.1849	0.2044	0.2214	0.2366	0.2505	0.2629	0.2737
II	0.5954	0.5881	0.5850	0.5830	0.5805	0.5765	0.5708
III	0.6771	0.6731	0.6697	0.6660	0.6603	0.6512	0.6382
IV	0.6771	0.6733	0.6711	0.6706	0.6718	0.6748	0.6789

NUMERICAL RESULT



FD is able to compute in the maximum field in the waveguide.

Compared the maximum value of the filed ridged waveguide with trapezoidal ridges and rectangular ridges.

One problem of the ridge waveguide is the reduced power capability.

CONCLUSION

A new FD approach to the computation of the modes of circular and elliptic waveguide has been described. Using an elliptical cylindrical grid, it takes exactly into account the curved boundary. Both TE and TM can be computed either on different grids or on the same grid.
The typical sparse matrix obtained by the FD allows an effective computation of the eigenvalues, with a very good accuracy, as shown by our tests.
 A further significant improvement in in the computational speed can be obtained using parallel architeture.

An irregular grid FD approach in the variable grid to the computation of the all modes of the waveguide has been described. The typical sparse matrix obtained by the FD allows an effective computation of the eigenvalues, with a very good accuracy, as shown by our tests visible on acts.
A for there significant improvement in in the computational speed can be obtained using parallel architeture.

THANK YOU FOR YOUR ATTENTION